Characterizing Rock Masses by the RMi for Use in Practical Rock Engineering

Part 1: The development of the Rock Mass index (RMi)

Arild Palmström

Present address: Norconsult International, Vestfjordgaten 4, N-1338 Sandvika, Norway

Abstract - The Rock Mass index, RMi, has been developed to satisfy a need for a strength characterization of rock masses for use in rock engineering and design. The method gives a measure of the reduction of intact rock strength caused by discontinuities given by $RMi = \sigma_c \times JP$ Here σ_c is the uniaxial compressive strength of the intact rock measured on 50 mm diameter samples, and JP is the jointing parameter which is a combined measure of block size (or intensity of jointing) and joint characteristics as measured by joint roughness, alteration and size. This paper describes the method of determining the RMi for a rock mass using various common field observations. The determination of a meaningful equivalent block size is a key issue which is discussed in detail. Several areas of application of the RMi are presented, among others for design of rock support. Discussion of these applications will be developed in Part 2 of this paper.

"The corner-stone of any practical rock mechanics analysis or rock engineering is the geological data base upon which the definition of rock types, structural discontinuities and material properties is based Even the most sophisticated analysis can become a meaningless exercise if the geological information upon which it is based is inadequate or inaccurate." Evert Hoek (1986).

1. Introduction

Rock masses may be considered as non-homogeneous construction materials built up of fragments and blocks of various sizes. As there is great diversity both in the composition of the intact rock and in the nature and extent of its discontinuities, rock masses exhibit a wider range in structure, composition and mechanical properties than most other construction materials. Reliable tests of the strength of such complex materials are impossible or so difficult to carry out with today's technology, that rock engineering pertaining to rock masses is currently based mainly on qualitative observational data. These qualitative observational data have to be expressed as numerical values to make calculations in rock engineering possible.

As the quality of the input data significantly affects accuracy in rock engineering and design, there is a need to improve the methods of rock mass description, and to develop practical guidelines for obtaining numerical observational data. The Rock Mass index (RMi) has been developed to satisfy this need.

The RMi is based on selected, well defined geological parameters. Existing methods for field description of outcrops, as well as logging of drill cores and geophysical measurements, have been refined. The RMi can be applied in various types of rock engineering with adjustment for features related to the particular project or utilisation of the rock. These applications are described in the second part to be published in the next issue.

2. The Rock Mass index (RMi)

Construction materials commonly used in civil engineering and mining are mostly characterized by their strength properties. This basic property of the material is used in the engineering and design. In rock engineering, no such specific strength characterization of the rock mass is in common use. Most engineering is carried out using various descriptions, classifications and unquantified experience. Hoek and Brown (1980), Bieniawski (1984), Nieto (1983) and several other authors have, therefore, indicated the need for a *strength characterization* of rock masses.

The Rock Mass index, RMi, has been developed to characterize the strength of the rock mass for construction purposes. An important issue has been to use parameters in the RMi, which have the greatest significance in engineering. This is discussed in detail in Section 4.1.



Figure 1 The main inherent parameters in the rock mass are applied in the RMi (from Palmström, 1995).

RMi applies only *intrinsic parameters* of the rock mass, see Figure 1. The need to use intrinsic parameters in characterizing rock masses has earlier been stressed by Patching and Coates (1968), among others.

RMi is based principally on the reduction in strength of a rock caused by jointing ¹ and is expressed as: $RMi = \sigma_c \times JP$ eq. (1)

where σ_c = the uniaxial compressive strength of intact rock measured on 50 mm samples;

JP = the jointing parameter which is a reduction factor representing the block size and the condition of its faces as represented by their friction properties and the size of the joints.

The influence of JP has been found using calibrations from test results. Because of problems of obtaining compression test results on rock masses at a scale similar to that of typical rock works, it was possible to find appropriate data from only eight large scale tests and one back analysis. These have been used to arrive at the following mathematical expression:

$$JP = 0.2\sqrt{jC} \times Vb^{D} \qquad eq. (2)$$

where Vb is given in m³, and $D = 0.37 \text{ jC}^{-0.2}$ has the following values:

		, L					5			0						
for	jC =	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	4	6	9	12	16	20
	D = (0.586	0.488	0.425	0.392	0.37	0.341	0.322	0.308	0.297	0.28	0.259	0.238	0.225	0.213	0.203

The value of JP varies from near 0 for crushed rocks to 1 for intact rock. The exponential form of eq. (2) fits well with the general experience that joint spacings have an exponential statistical distribution as shown by Merritt and Baecher (1981).

The joint condition factor is expressed as jC = jL (jR/jA) where jL, jR and jA are factors for respectively, joint length and continuity, joint wall roughness, and joint surface alteration. Their ratings are shown in

¹ The term 'joint' has been used for most natural discontinuities, which have thickness smaller than approx. 0.1 m. Thus, joints cover fissures, partings, fractures, natural cracks, as well as many shears and seams.

Tables 1 to 3. The factors jR and jA are similar to the joint roughness number (Jr) and the joint alteration number (Ja) in the Q-system.² The joint size and continuity factor (jL) has been introduced in the RMi system to represent the scale effect of the joints.

Most commonly, the joint condition factor jC = 1 to 2; thus, the jointing parameter will vary between $JP = 0.2 \text{ Vb}^{0.37}$ and $JP = 0.28 \text{ Vb}^{0.32}$. For jC = 1.75 the jointing parameter can simply be expressed as:

$$JP = 0.25 \sqrt[3]{Vb}$$
 eq. (2a)

Significant *scale effects* are generally involved when the tested rock volume is enlarged from laboratory size to field size. From the calibration described above, the RMi is tied to large samples where the scale effect has be included in JP. For massive rock masses, however, the scale effect for the uniaxial compressive strength (σ_c) has not been accounted for, as σ_c is related to 50 mm sample size. As shown in Figure 2, Barton (1990) suggests from data presented by Hoek and Brown (1980) and Wagner (1987), that the actual compressive strength for large 'field samples' may be determined from:

 $\sigma_{cf} = \sigma_{c50} (0.05/Db)^{0.2} = \sigma_{c50} \times f_{\sigma}$ eq. (3)

where σ_{c50} = the uniaxial compressive strength for 50 mm sample size,

Db = block diameter measured in metre,

 $f_{\sigma} = (0.05/Db)^{0.2}$ is the scale factor for compressive strength.



Figure 2. Empirical equations for the scale effect of uniaxial compressive strength (from Barton (1990), based on data from Hoek and Brown, 1980 and Wagner, 1987).

Eq. (3) is valid for sample diameters up to some metres, and may, therefore, be applied for massive rock masses as indicated in Figure 2. The equivalent block diameter (Db) may be found from $Db = \sqrt[3]{Vb}$ or, in cases where a pronounced joint set occurs, from Db = S, where S is the spacing of this set. If the block shape factor (β) is known (see Appendix, Sections A5 and A6) the equivalent block diameter is:

$$Db = \frac{\beta_o}{\beta} \sqrt[3]{Vb} = \frac{27}{\beta} \sqrt[3]{Vb} \qquad \text{eq. (4)}$$

In addition to the block shape factor, the Appendix describes various types of measurements, which can be used to estimate the block volume.

Table 1. The ratings of the joint roughness factor, (jR, found from smoothness and waviness (from Palmström, 1995) (The ratings of jR are similar to Jr in the Q-system)

 $^{^{2}}$ The symbols Jr and Ja have been changed into jR and jA because some minor modifications have been made in their definitions.

small scale	large scale waviness of joint plane					
smoothness of joint surface	planar	slightly undulating	strongly undulating	stepped	interlocking (large scale)	
very rough	3	4	6	7.5	9	
rough	2	3	4	5	6	
slightly rough	1.5	2	3	4	4.5	
smooth	1	1.5	2	2.5	3	
polished	0.75	1	1.5	2	2.5	
slickensided ^{*)}	0.6 - 1.5	1 - 2	1.5 - 3	2 - 4	2.5 - 5	
	For filled joints:	jR = 1	For <u>irregular joints</u> a ra	ating of $jR = 5$ is	suggested	

*) For slickensided joints the value of jR depends on the presence and appearance of the striations; the highest value is used for marked striations.

Table 2. Characterization and rating of the joint alteration factor, jA, (from Palmström, 1995) (jA is similar to Ja in the Q-system, except for the grade of alteration)

A. CONTACT BETWEEN THE TWO ROCK WALL SURFACES					
TERM	DESCRIPTION		jA		
Clean joints -Healed or "welded" joints -Fresh rock walls -Alteration of joint wall: · 1 grade more altered · 2 grades more altered	Softening, impermeable filling (quartz, epidote etc.) No coating or filling on joint surface, except for stain The joint surface exhibits one class higher alteration The joint surface shows two classes higher alteration	0.75 1 2 4			
Coating or thin filling -Sand, silt, calcite etc. -Clay, chlorite, talc etc.	Coating of friction materials without clay Coating of softening and cohesive minerals	3 4			
B. FILLED JOINTS WIT	H PARTIAL OR NO CONTACT BETWEEN THE	ROCK WALL	SURFACES		
TYPE OF FILLING MATERIAL	DESCRIPTION	Partial wall contact thin fillings (< 5 mm ^{*)}) jA	No wall contact thick filling or gouge jA		
-Sand, silt, calcite etc. -Compacted clay materials -Soft clay materials -Swelling clay materials	Filling of friction materials without clay "Hard" filling of softening and cohesive materials Medium to low over-consolidation of filling Filling material exhibits clear swelling properties	4 6 8 8 - 12	8 10 12 12 - 20		

*) Based on joint thickness division in the RMR system (Bieniawski, 1973)

Table 3. The joint size and continuity factor, jL, (from Palmström, 1995).

JOINT LENGTH	TERM	TYPE	continuous joins	jL discontinuous joints ^{**)}
< 0.5 m	very short	bedding/foliation partings	3	6
0.1 - 1.0 m	short/small	joint	2	4
10 - 30 m	long/large	joint	0.75	1.5
> 30 m	very long/large (filled	l) joint, seam ^{*)} or shear ^{*)}	0.5	1

*) Often occurs as a single discontinuity, and should in these cases be treated separately. *** Discontinuous joints end in massive rock



Figure 3. The jointing parameter (JP) found from the joint condition factor (jC) and various measurements of jointing intensity (Vb, Jv, RQD). The determination of JP from Vb (or RQD or Jv) in the examples are indicated (from Palmström, 1995).

Figure 3 shows how the jointing parameter (JP) can be found from the block volume (Vb) and the joint condition factor (jC). As shown in the upper left part of the diagram, the volumetric joint count (Jv) for various joint sets (and/or block shapes) can be used instead of the block volume, see Appendix. Also, the RQD can be used, but its inability to characterise massive rock or highly jointed rock masses leads to a reduced quality of JP.

The classification of RMi is presented in Table 4. Numerical values alone are seldom sufficient for characterizing the properties of a complex material such as a rock mass. Therefore, the RMi and its parameters should be accompanied by supplementary descriptions.

5

for RMi	related to rock mass strength	RMi VALUE
Extremely low	Extremely weak	< 0.001
Very low	Very weak	0.001 - 0.01
Low	Weak	0.01 - 0.1
Moderate	Medium	0.1 - 1
High	Strong	1 - 10
Very high	Very strong	10 - 100
Extremely high	Extremely strong	> 100
Extremely high	Extremely strong	

Table 4. Classification of RMi (from Palmström, 1995)

3. Examples

The values of the jointing parameter (JP) found in the following examples are also shown in Figure 3.

Example 1

The block volume has been measured as $Vb = 0.003 \text{ m}^3$ (= 3 dm³). As given in Tables 1 to 3, the joint condition factor jC = 0.75 is determined from:

- the rough joint surfaces and small undulations of the joint wall which gives jR = 3;

- the clay coated joints, i.e. jA = 4; and

- the 3 - 10 m long, continuous joints, which gives jL = 1.

Applying the values for Vb and jC in Figure 5, a value of JP = 0.02 is found.ⁱ⁾ With a compressive strength of the rock $\sigma_c = 150$ MPa, the value of RMi = $0.02 \cdot 150 = 3$ (high). ⁱ⁾ using eq. (2) a value of JP = 0.018 is found

$using eq. (2) u value of 51 0.010 is_{j}$

Example 2

The block volume $Vb = 0.6 \text{ m}^3$. The joint condition factor jC = 2 is determined from Tables 1 to 3, based on:

- smooth joint surfaces and planar joint walls which gives jR = 4;

- fresh joints, jA=1; and 1 - 3 m long discontinuous joints, i.e. jL=3.

From Figure 5 the value JP = 0.25 is found.ⁱⁱ⁾ With a compressive strength $\sigma_c = 50$ MPa of the rock, the value of RMi = 12.5 (very strong).

ⁱⁱ⁾ JP = 0.24 is found using eq. (2)

Example 3

Values of RQD = 50 and jC = 0.2 give JP = 0.007

Example 4

Two joint sets spaced 0.3 m and 1 m, and some random joints have been measured. The volumetric joint count is $Jv = 1/0.3 + 1/1 + 0.5^{iv} = 4.5$

With a joint condition factor jC = 0.5 the jointing parameter JP = 0.12 (by using the column for 2 to 3 joint sets in Figure 5)

iv) the assumed value for the random joints

Example 5

The following jointing features are measured: one joint set with spacing S = 0.45 m, and a joint condition factor jC = 8. For this massive rock it is seen in Figure 3 that the value of JP is determined from the scale factor for compressive strength $f_{\sigma} = 0.45$. For a rock with $\sigma_c = 130$ MPa the value of RMi = 59.6 (very strong).

4. Discussion

4.1 On the Selection of the Parameters Used in RMi

As shown in Figure 1, the RMi makes use of the following input parameters: compressive strength of intact rock, block volume, and joint characteristics as given by roughness, alteration and size. In addition to the author's own experience, the study of some 15 different classification systems have been made for the selection of these input parameters.

Hoek et al. (1992), is of the opinion that the strength characteristics for jointed rock masses are controlled by the block shape and size as well as their surface characteristics determined by the intersecting joints. They recommend that these parameters be selected to represent the average condition of the rock mass. Also, Tsoutrelis et al. (1990), Matula and Holzer (1978), Patching and Coates(1968) and Milne et al. (1992) have been set forth similar ideas. This does not imply that the properties of the intact rock material should be disregarded in the characterization. If joints are widely spaced or if the intact rock is weak, the properties of the intact rock may strongly influence the overall behaviour of the rock mass. The intact rock properties are also important if the joints are discontinuous.

Although rock mass properties in many cases are governed by joints, rocks properties have been a major factor in the formation and development of the actual joints. In this respect petrological data can make an important contribution towards the prediction of mechanical performance of the jointing features (Franklin, 1970). For this reason, it is important to retain the names for the different rock types, for these in themselves give relative indications of the joint properties (Piteau, 1970). A supplementary rock description will also inform the reader of the geology and the type of material at the site.

4.2 Benefits and Limitations of RMi

Some of the benefits of the RMi system are:

- *The RMi will give significant improvements in the use of geological input data,* mainly achieved by its systematic use of well defined parameters in which the three-dimensional character of rock masses is represented by the block volume.
- *The RMi can easily be used for rough estimates when limited information on the ground conditions is available.* For example, in early stages of a project where rough estimates are sufficient, eq. (2a) can be applied.
- *The RMi is well suited for comparisons and exchange of knowledge between different locations.* In this way it may contribute to improved communication between people involved in rock engineering and design.
- *The RMi offers a platform suitable for engineering judgement.* RMi is a general parameter which characterizes the inherent strength of rock masses, and may be applied in engineering as the quality for this construction material. As RMi is composed of real block volumes and common joint parameters for rock masses, it is easy to relate it to field conditions. This is important in application of engineering judgement.
- *The RMi system covers a wide spectrum of rock mass variation,* and therefore has possibilities for wider applications than other rock mass classification and characterization systems of today.

Any attempt to mathematically express the variable structure and properties of jointed rock masses in a general failure criterion, may result in complex expressions. By restricting the RMi to uniaxial compressive strength only, it has been possible to arrive at the relatively simple expressions in eqs. (1) and (2). Because simplicity has been preferred in the structure as well as in the selection of parameters in RMi, it is clear that such an index may result in inaccuracy and limitations, of which the main are connected to:

• The range and types of rock masses covered by the RMi. Both the intact rock material as well as the joints exhibit great directional variations in composition and structure which results in a large range in compositions and properties of rock masses. It is, therefore, not possible to characterize all these

combinations in one, single number. Nevertheless, the RMi probably characterizes a wider range of materials than most classification systems.

- *The accuracy in the expression of RMi.* The value of the jointing parameter (JP) is calibrated from a few large scale compression tests. Both the evaluation of the various factors (jR, jA and Vb) used in obtaining JP and the size of the samples tested, which in some of the cases had a small number of blocks, may be sources of error in the expression for JP. The value of RMi found may therefore be approximate. In some cases, however, errors in the various parameters may partly neutralize each other.
- *The effect of combining parameters that vary in range.* The parameters used to calculate the RMi will in general *express a certain range of values.* As with any classification system, combination of such variables may cause errors. In some cases the result is that the RMi may be inaccurate in its characterization of the strength of the complex and varied assemblage of the materials and defects which constitute a rock mass. For these reasons, the RMi may best be considered as a *relative* index in its characterization of the rock mass strength.

4.3 Other Similar Rock Mass Characterization Methods

A similar approach to a strength characterization of rock masses has been proposed by Hansagi (1965, 1965b), who introduced a reduction factor (C_g) comparable to the jointing parameter (JP) to arrive at an expression for the *compressive strength* of the rock mass, expressed as

Hansagi named C_g as a "gefüge-factor" (joint factor) that is "*representative for the jointed effect of a rock mass*". This factor consists of two inputs: a factor for the "structure of jointing" (core length), and a scale factor. Hansagi (1965b) mentions that the value of C_g is 0.7 for massive rock and 0.47 for jointed rock (containing small joints) for two test locations in Kiruna, Sweden. Hansagi did not, however, as far as the author knows, publish more on his method.

The expression for the RMi is similar in structure to the expression of unconfined *compressive strength of rock masses* (σ_{cm}), which is a part of the Hoek-Brown failure criterion for rock masses expressed as

$$\sigma_{\rm cm} = \sigma_{\rm c} \times s^{\frac{1}{2}} \qquad \qquad {\rm eq.} \ (12)$$

Here s = an empirical constant. The value of s ranges from 0 for jointed rock masses to 1 for intact rock.

The value of *s* is found using the RMR or the Q classification system as described by Hoek (1983), Hoek and Brown (1980, 1988), and Wood (1991). This constant is more accurately found from JP than via these classification systems. RMi introduces an easier and more direct method to find the values of $s (= JP^2)$ as JP involves only inherent features in the rock mass which have a direct impact on *s*. In this way, RMi may contribute to a future improvement of the Hoek Brown failure criterion.

4.4 Possible Applications of the RMi

The main purpose during development of the RMi has been to work out a practical system to characterize rock masses which is applicable to rock engineering and design. When applied, the RMi-value or its parameters are adjusted for local features of importance for the engineering purpose, see Figure 4.

Figure 5 shows the main areas for application of RMi together with the influence of its parameters in different fields. Some of these will be shown in a subsequent paper which will outline the practical use of RMi.

The RMi-value can seldom be used directly in classification systems as many of them are systems made for a particular purpose. Some of the input parameters in RMi are sometimes similar to those used in the classifications and may then be applied more or less directly.



Finally, the system for characterizing block geometry (volume, shape factor, angles) may be of use in numerical models.

Figure 4. The principle application of RMi in rock engineering (from Palmström, 1995).



Figure 5. The main applications of RMi in rock mechanics and rock engineering (from Palmström, 1995).

Acknowledgement

This paper is part of a Ph.D. thesis titled "RMi - A rock mass characterization system for rock engineering purposes" which has been made at the University of Oslo, Norway. The funding by the Norway Research Council (NFR) has made this work possible. I am most grateful for all support from the Norwegian Geotechnical Institute during my studies and to Ole Berthelsen for valuable comments.

5. References

Barton, N., Lien, R. and Lunde, J. (1974):
Engineering classification of rock masses for the design of rock support.
Rock Mechanics 6, 1974, pp. 189-236.
Barton N. (1990):
Scale effects or sampling bias?
Proc. Int. Workshon Scale Effects in Rock Masses Balkema Publ. Rotterdam. pp. 31-55
Rienjawski 7 T (1973).
Engineering elessification of jointed rock masses
Trans S. African Instr. Civ. Engrs. Vol. 15. No. 12. Dec. 1073, pp. 335 - 344
Diamiourski 7 T. (1094).
Dieliidwski Z. I. (1964).
Rock mechanics design in mining and tunneling.
A.A. Balkema, Rotterdam, 2/2 pp.
Dearman W.R. (1991):
Engineering geological mapping.
Butterworth - Heinemann Ltd., Oxford.
Franklin J.A. (1970):
Observations and tests for engineering description and mapping of rocks.
Proc. 4th Congress ISRM, Beograd 1970, 6 pp.
Hansagi I. (1965):
Numerical determination of mechanical properties of rock and of rock masses.
Int. J. Rock Mech. Mining Sci., Vol 2, pp. 219-223.
Hansagi I. (1965b):
The strength properties of rocks in Kiruna and their measurements (in Swedish).
Ingenjörsvetenskapsakademiens meddelande IVA no. 142. Stockholm pp. 128-143
Hoek E and Brown E T (1980):
Underground excavations in rock
Institution of Mining and Metallurgy London 1980, 527 pp
Hook E (1086).
Dreatical reals machanica development over the next 25 years
Vernete address delivered 24.2 1096
Keynole address delivered 24.2.1980
Hoek E. and Brown E.T. (1988):
The Hoek-Brown failure criterion - a 1988 update.
Proc. 15th Canadian Rock Mechanics Symp. 1988, pp. 31-38.
Hoek E., Wood D. and Shah S. (1992):
A modified Hoek-Brown failure criterion for jointed rock masses.
Proc. Int. Conf. Eurock '92, Chester, England, pp. 209-214.
Matula M. and Holzer R. (1978):
Engineering topology of rock masses.
Proc. of Felsmekanik Kolloquium , Grundlagen und Andwendung der Felsmekanik, Karlsruhe
Germany, 1978, pp. 107-121.
Merritt A.H. and Baecher G.B. (1981):
Site characterization in rock engineering.
22nd U.S. Symp. on Rock Mechanics, pp. 49-66.
Milne D., Germain P. and Potvin Y. (1992):
Measurement of rock mass properties for mine design.
Proc. Int. Conf. Eurock '92. Thomas Telford, London, pp. 245-250.
Nieto A S (1983):
Some geologic factors in the location design and construction of large underground chambers in
rock Proc Rapid Excavation & Tunneling Conf. AIME 1983, pp. 569-596
Palmetröm Δ (1082).
The volumetric joint count - a useful and simple measure of the degree of jointing
Prog. IV Int. Congr. IAEG. Now Dolb; 1092 nr V 221 V 229
1100. IV IIII. CONSI. IADO, INCW DEIIII, 1902, pp V.221-V.220.

Palmström A. (1985): Application of the volumetric joint count as a measure of rock mass jointing. Proc. Int. Symp. Fundamentals of Rock Joints, Björkliden, Sweden, 1985, pp 103-110. Palmström A. (1986): The volumetric joint count as a measure of rock mass jointing. Presented at the Conference on Fracture, Fragmentation and Flow, Jerusalem 1986, 19 pp. Palmström A. (1995): RMi - a rock mass characterization system for rock engineering purposes. Ph.D. thesis, University of Oslo, Norway, 400 pp. Patching T.H. and Coates D.F. (1968): A recommended rock classification for rock mechanics purposes. CIM Bull., Oct. 1968, pp. 1195-1197. Piteau D.R. (1970): Geological factors significant to the stability of slopes cut in rock. Proc. Symp. on Planning Open Pit Mines, Johannesburg, South Africa, 1970, pp. 33-53. Selmer-Olsen R. (1964): Geology and engineering geology. (in Norwegian) Tapir publishing firm, Trondheim, Norway, 409 pp. Sen Z., Eissa E.A. (1991): Volumetric rock quality designation. J. Geotech. Engn., Vol 117, No 9, 1991, pp 1331 - 1346. Sen Z. and Eissa E.A. (1992): Rock quality charts for log-normally distributed block sizes. Int. J. Rock Mech. Min. Sci. & Geomech. Abstr., Vol. 29, No. 1, pp. 1-12. Terzaghi R. (1965): Sources of error in joint surveys. Geotechnique, Vol 15, 1965, pp 287-304. Tsoutrelis C.E., Exadactylos G.E. and Kapenis A.P. (1990): Study of the rock mass discontinuity system using photoanalysis. Proc. from Symp. on Mechanics of Jointed and Faulted Rock, 1990, pp.103-112. Wagner H. (1987): Design and support of underground excavations in highly stressed rock. Proc. 6th ISRM Congr., Montreal; Keynote paper Vol. 3. Williamson D.A. and Kuhn C.R (1988): The unified classification system. Rock Engineneering Systems for Engineering Purposes, ASTM STP 984, American Society for Testing Materials, Philadelphia, pp. 7 - 16. Wood D. (1991): Estimating Hoek-Brown rock mass strength parameters from rock mass classifications.

Transportation Research Record 1330, pp. 22-29.

A P P E N D I X

METHODS AND CORRELATIONS TO DETERMINE THE BLOCK VOLUME

"The purpose of science is to simplify, not to complicate. The function of an engineering geologist, geotechnical or rock engineer is to examine and observe the complex variables of an area or project site and from this effort arrive at a set of simple, significant generalizations." Williamson D.A and Kuhn C.R. (1988)

A1. Introduction

The block size is usually the most important factor in the RMi. Consequently, the accuracy of this measure has a significant impact on the quality of the RMi. This appendix presents methods to determine the block volume from various types of jointing observations and measurements.

The block size is a result of the detailed (small to medium scale) jointing in a rock mass formed mainly by the small and moderate joints (Selmer-Olsen, 1964). The block dimensions are determined by joint spacings and the number of joint sets. Individual or random joints and possibly other planes of weakness may further influence the size and shape of blocks. Impact from rock blasting may also be an influence.

Different methods have been developed over the years to measure the quantity or density of joints in the rock mass. The selection of the method(s) to be applied at a particular site is often a result of the:

- a) availability of exposures to observe the rock and its jointing,
- b) quality requirements for the collected data,
- c) the type and cost of the investigation or survey, and
- d) the experience of the engineering geologist.

If all the blocks in a rock mass could be measured or "sieved", a block size distribution can be found similar to the particle size distribution of a soil. As the joint spacings generally vary greatly, the difference in size between the smaller and the larger blocks can be significant, see Figure A1. Therefore, the characterization of block volume should be given rather as an interval than as a single value. The relationship between block volume and soil size particle is outlined in Table A1.



Figure A1. Example of a block size distribution curve for a rock mass (from Milne et al., 1992)

If less than 3 joint sets occur defined blocks may not be found. However, in many cases the presence of random joints or other weakness planes may contribute to defining blocks. Where the jointing is irregular, or many of the joints are discontinuous, it can sometimes be difficult to recognize the actual size and shape of individual blocks. Thus, from time to time the block size and shape therefore have to be determined using a simplification where an *equivalent block volume* is used as is described in Section A9. *Table A1. Classification of block volume related to particle size (volume) for soils (from Palmström, 1995)*

TERM FOR	TERM FOR	Block	TERM FOR	Approx.
DENSITY	BLOCK SIZE	volume ^{*)}	SOIL	particle
OF JOINTS		(Vb)	PARTICLE vo	olume ^{*)}
			Coarse sand	$0.1 - 5 \text{ mm}^3$
			Fine gravel	$5 - 100 \text{ mm}^3$
Extremely high	Extremely small	< 10 m ³	Medium gravel	$0.1 - 5 \text{ cm}^3$
Very high	Very small	$10 - 200 \text{ m}^3$	Coarse gravel	$5 - 100 \text{ cm}^3$
High	Small	0.2 - 10 m ³	Cobbles	$0.1 - 5 \mathrm{dm}^3$
Moderate	Moderate	$10 - 200 \text{ m}^3$	Boulders	5 - 100 dm ³
Low	Large	0.2 - 10 ³	Blocks	$> 0.1 \text{ m}^3$
Very low	Very large	$10 - 200 \text{ m}^3$		
Extremely low	Extremely large	$> 200 \text{ m}^3$		

 $^{*)}$ Vb = 0.58 Db³ has been applied in the correlation between particle diameter and particle or block volume.

Table A2. The main types of observations and measurements which can be used to estimate the degree of jointing and the block size (from Palmström, 1995)

	TYPE OF MEASUREMENT						
PARAMETER MEASURED	Surface of	Drill core or scanline observations					
	3-D observations	2-D observations	1-D observations				
BLOCK SIZE	Block volume estimated from defined joint spacings (and angles between joint sets)						
- Block volume	Block volume estimated from the volumetric joint count (Jv)						
	Block volume measured directly in the field.		Fragment volume found in drill cores ¹⁾ .				
- Equivalent block diameter		Estimated block diameter (Ib) according to ISRM (1978).	Indirect block diameter measure (given as RQD).				
DEGREE OF JOINTING		Measured number of joints intersecting an area.	Measured number of joints intersecting a line.				
- Joint	Observation of the volumetric joint count (Jv).	*Weighted joint density measurement.	*Weighted joint density measurement.				
nequency			Density of joints estimated from refraction seismic velocities. ²⁾				
- Joint spacing	Measured spacings for each joint set.	Measured average joint spacings related to a plane.	Measured length of core pieces along a bore hole (fracture inter- cept (ISRM, 1978))				

* Measurement method introduced by Palmström (1995), see Section A8.

¹⁾ The particle volume referred to has the size of core diameter or less (gravel or pebble size)

²⁾ Not included in this paper

Observations made on surfaces or on drill cores are most commonly used to characterize the density or amount of joints in a rock mass. The methods most commonly used are shown in Table A2. As the blocks generally have varying sizes and shapes, the measurements of characteristic dimensions can be time-consuming and laborious. To remedy this, easy recognizable dimensions of the blocks and simple correla-

tions between the different types of jointing measurements have been preferred, as is shown in this Appendix.

A2. Block Volume Measured Directly In-Situ or in Drill Cores

Where the individual blocks can be observed in a surface, their volume can be directly measured from relevant dimensions by selecting several representative blocks and measuring their average dimensions. For small blocks or fragments having volumes in dm³ size or less, this method of block volume measurement is often beneficial as it is much easier to estimate volume compared to all the measurements which have to be made to include all joints. Block volume can also be found in drill cores where small fragments have been formed as a result of crushed rock.

A3. Block Volume Found from Joint Spacings

The terms *joint spacing* and *average joint spacing* are often used in the description of rock masses. Joint spacing is the distance between individual joints within a joint set. Where more than one set occurs, this measurement is, in the case of surface observations, often given as the average of the spacings for these sets. There is often some uncertainty as to how this average value is found; for instance, the average spacing for the following 3 joint sets having spacings S1 = 1 m, S2 = 0.5 m, and S3 = 0.2 m is Sa = 0.125 m, and not 0.85 m which initially may seem appropriate.³

When logging drill cores the average length of core pieces⁴ or frequencies are seldom true spacings, as joints of different sets probably are included in the measurement. In addition, random joints which do not necessarily belong to any joint set, have an influence.

As the term 'joint spacing' does not indicate what it includes, it is frequently difficult to determine whether a 'joint spacing' referred to in the literature represents the true joint spacing. Thus, there is often much confusion related to joint spacing recordings.

Especially where irregular jointing occurs, it is time-consuming to measure all (random) joints in a joint survey. In such cases, as well as for other jointing patterns, it is often much quicker - and also more accurate - to measure the block volume directly in the field. Where three regular joint sets occur, the block volume can easily be found from the joint spacings as

$$Vb = \frac{S1 \times S2 \times S3}{\sin \gamma 1 \times \sin \gamma 2 \times \sin \gamma 3} = \frac{Vb_0}{\sin \gamma 1 \times \sin \gamma 2 \times \sin \gamma 3}$$
eq. (A-1)

where $\gamma 1, \gamma 2, \gamma 3$ are the angles between the joint sets, and

S1, S2, S3 are the spacings between the individual joints in each set.

 Vb_0 is the block volume in cases where joints intersect at right angles.

For a rhombohedral block with two angles between 45° and 60° , two between 135° and 150° and the last two being 90° , the volume will be between Vb = 1.3 Vb_o and 2 Vb_o. Compared to the variations caused by the joint spacings, the effect from the intersection angle between joint sets is relatively small.

A4. Block Volume Found from Joint Frequency Measurements

When the frequency is given for each joint set, it is possible to find the block volume directly. In other cases, when an 'average frequency' is given, it is uncertain whether this frequency value refers to one-, two- or three-dimensional measurements; hence no accurate correlation can be presented. The use of joint

³ The average spacing is found from 1/Sa = 1/S1 + 1/S2 + 1/S3

⁴ Joint or fracture intercept is the appropriate term for measurement of the distance between joints along a line or bore hole.

frequency measurements presented in the following are similar to the joint spacing measurements shown in Section A3.

A4.1 From 2-D Joint Frequency Measurements on an Area or Surface

The 2-D joint frequency is the number of joints measured in an area. The length of the joints compared to the size of the area will, however, influence on the frequency observed. Thus, some sort of adjustments have to be made to estimate the block volume from this type of measurement. The joint frequency (Na) found in an observation surface, therefore, should be adjusted for the lengths of the joints if they are shorter than the length of the observation plane, expressed as

$$Na = (1/\sqrt{A}) \Sigma(na_i \cdot L_i) + Na_i \qquad eq. (A-2)$$

where $na_i = the joint i$ with length L_i shorter than the length of the observation area, $Na_j = the number of joints longer than the length of the observation area, and$ <math>A = the area of the observation surface.

The joint frequency (Na) varies with the orientation of the observation plane and with respect to the attitude of the joints. Recording of Na in several surfaces of various orientation gives a more accurate measure of the jointing. Being an average measure, Na should be measured in selected areas showing the same type and density of jointing. Thus, a large area should be divided into smaller, representative areas containing similar jointing, and the variation in jointing for the whole area calculated based on these observations.



Figure A2 The connection between 2-D joint observations in a surface and the volumetric joint count (Jv) for various jointing patterns and orientations of the observation plane (from Palmström, 1995).

eq. (A-3)

The correlation between 2-D measurements of the joint density in a rock surface and the 3-D frequency values (given as Jv) can be done using the empirical expression

 $Jv = Na \times ka$

where ka = correlation factor shown in Figure A2; ka varies mainly between 1 and 2.5 with an average value ka = 1.5. The highest value is where the observation plane is parallel to the main joint set.

A4.2 From 1-D Jointing Frequency Measurements along a Scanline or Drill Core

This is a record of the joint frequency along a bore hole or a scanline given as the number of joints intersecting a certain length. This 1-D joint frequency is an average measure along the selected length of the

core. As in other core logging methods, it is important to measure the joints in sections along the line or core which shows similar joint frequency. At the start of the logging it is rational to divide the length into such sections.



Figure A3 The relationship between 1-D joint observations in drill cores or along a scanline and the volumetric joint count (Jv) for various jointing patterns and orientations of the bore hole (or scanline) (from Palmström, 1995).

The correlation between 1-D joint frequency observations in drill holes (or scanlines) and volumetric 3-D frequency (Jv) can be done using an expression similar to eq. (A-10). The joint frequency, given as the number of joints per metre, can be expressed as:

 $Jv = Nl \times kl$

eq. (A-4)

where kl = correlation factor. As shown in Figure A3 kl varies between 1.25 and 6, with an average value kl = 2. As expected there is a rather poor correlation between Jv and Nl.

A5. Block Volume Calculated from the Volumetric Joint Count

A5.1 The Volumetric Joint Count (Jv)

The volumetric joint count (Jv) has been described by Palmström (1982, 1985, 1986) and Sen and Eissa (1991, 1992). It is a measure of the number of joints within a unit volume of rock mass, defined by

 $Jv = \Sigma (1/S_i)$ eq. (A-5a) where $S_i =$ the joint spacing in metres for the each joint set *i*.

Also random joints can be included by assuming a random spacing for each of these. Experience indicates that this can be set to Sr = 5 m; thus, the volumetric joint count can be generally expressed as

 $Jv = \Sigma (1/S_i) + Nr/5$ eq. (A-5b)

where Nr = the number of random joints. A more accurate determination of Nr can be found applying a method similar to that described for na_i in eq. (A-2).

Jv can easily be calculated from joint observations, since it is based on measurements of joint spacings or frequencies. In the cases where mostly random or irregular jointing occur, Jv can be found by counting all the joints observed in an area of known size as described in Section A4.1.

Since both the volumetric joint count (Jv) and the size of blocks in a rock mass vary according to the degree of jointing, there is a correlation between them (Palmström, 1982). Jv varies with the joint spacings, while the block size also depends on the type of block. A correlation between the two parameters has therefore to be adjusted or corrected for the block shape and the angle between the joint sets, as shown below.

The volumetric joint count determined from three joint sets with intersecting angles $\gamma 1$, $\gamma 2$ and $\gamma 3$ is expressed as

$$J_{V} = \frac{S2 \times S3 + S1 \times S3 + S1 \times S2}{Vb \times \sin \gamma 1 \times \sin \gamma 2 \times \sin \gamma 3}$$
eq. (A-6)

where S1, S2, S3 are the joint spacings.

From eq. (A-4b) the block volume is

$$Vb = \beta \times J_V^{-3} \frac{1}{\sin \gamma 1 \times \sin \gamma 2 \times \sin \gamma 3}$$
 eq. (A-7)

Using $Vb_0 = Vb \cdot \sin\gamma 1 \times \sin\gamma 2 \times \sin\gamma 3$ the block volume is, for cases where all angles between the block faces are 90°, given as

$$Vb_o = \beta \times Jv^{-3}$$
 eq. (A-7a)

The factor

$$\beta = \frac{(\alpha 2 + \alpha 2 \times \alpha 3 + \alpha 3)^3}{(\alpha 2 \times \alpha 3)^2}$$
 eq. (A-8)

(where $\alpha 2 = S2/S1$ and $\alpha 3 = S3/S1$) depends mainly on the differences between the joint set spacings. It has therefore been named the *block shape factor* and is further described in Section A6.

As the volumetric joint count (Jv) by definition takes into account in an unambiguous way all the occurring joints in a rock mass, it is often appropriate to use Jv in the correlation between joint frequency measurements and block volume estimates (Palmström, 1982). Important here is the block shape factor β which is included in all equations to estimate the block volume.

A6 Block Types and Shapes

Methods to determine the block shape factor β given in eq. (A-8) and its characterization are described in this section. The *type and shape* of blocks are determined by:

- the number of joint sets;
- the differences in joint spacings; and
- the angles between the joints or joint sets.

For a rock mass with 3 joint sets intersecting at right angles the values of β are given Figure A.4. The types of blocks delineated by joints have in the literature been characterized in different ways and by different terms. Where relatively regular jointing exists, it may be possible to give adequate characterization of the jointing pattern according to the system presented by Dearman (1991). In most cases, however, there is no regular jointing pattern; a rough characterization of the blocks is generally more practical, for example a division into three main groups only, as presented by Sen and Eissa (1991). The terms applied by Palmström



(1995) are shown in Figure A4. For $\beta = 27$ to 32 the block term 'compact' has been introduced to include cubical, equidimensional, blocky and other existing terms for blocks not being long or flat.

Figure A4. Block types characterized by the block shape factor (β) found from the ratio between spacings of the joint sets. The data are based on 3 joint sets intersecting at right angles (from Palmström, 1995). Example: For $\alpha 2 = 4$ and $\alpha 3 = 15$, $\beta = 135$.

The use of Figure A3 requires 3 joint sets. As blocks often have more than six faces or have irregular shape, it can be difficult to estimate β . Therefore, the following simplified method to estimate β has been developed by Palmström (1995), in which the longest and shortest dimension of the block are applied:

$$\beta = 20 + 7 a^3/a^1 = 20 + 7 \alpha^3$$
 eq. (A-9)

where a3 and a1 are the longest and shortest dimension of the block.

The evaluations made by Palmström (1995) have shown that eq. (A-9) covers most types of blocks (where β < 1000) within reasonable accuracy (± 25%). For very flat to extremely flat blocks eq. (A-9) has limited accuracy.

A7. A Correlation between RQD and the Volumetric Joint Count (Jv)

It is not possible to obtain good correlations between RQD and Jv or between RQD and other measurements of jointing. Palmström (1982) presented the following simple expression:

RQD = 115 - 3.3 Jv

eq. (A-10)

Here RQD = 0 for Jv > 35, and RQD = 100 for Jv < 4.5

Especially where many of the core pieces have lengths around 0.1 m, the correlation above may inaccurate. However, when RQD is the only joint data available, eq. (A-10) has been found to be the best simple transition from RQD via Jv to block volume.

The block volume can be found from the volumetric joint count using input of the block shape factor (β) (see eq. (A-7) and (A-7a)). Where β is not known, it is recommended to use a 'common' value of $\beta = 40$.

A8 Weighted Joint Density Measurements (wJd)

R. Terzaghi (1965) points out that the accuracy of jointing measurements can be increased by replacing the number of joints measured on a surface or in a bore hole (N_{α}) intersected at an angle α , by a value N_{90} . N_{90} represents the number of joints with the same orientation which would have been observed at an intersection angle of 90°. This is expressed as

$$N_{90} = N_{\alpha} / \sin \alpha$$

eq. (A-11)

Terzaghi stresses the problem connected to small values of α , because, in these cases, the number of intersections will be significantly affected by local variations in spacing and continuity. "No correction whatsoever can be applied if α is zero. Hence N_{90} would fail to correctly indicate the abundance of horizontal and gently dipping joints in a horizontal observation surface."

The weighted joint density method is based on measuring the intersection angle between each joint and the observation surface or bore hole. To solve the problem of small intersection angles and to simplify the observations, the angles have been divided into intervals as shown in Table A3.



Figure A5. The intersection between joints and a drill core hole (left) and a surface (right) (from *Palmström*, 1995).

For 2-D measurements (surface observations) the weighted joint density is defined as

wJd =
$$(1/\sqrt{A}) \Sigma(1/\sin\delta_i) = (1/\sqrt{A}) \Sigma(f_i)$$
 eq. (A-12)

and, similarly, for 1-D measurements along a scan line or in drill cores

wJd =
$$(1 / L) \Sigma(1/\sin \delta_i) = (1 / L) \Sigma(f_i)$$
 eq. (A-13)

where δ_i = the angle between the observation plane (surface) and the individual joint.

A = the size of the area in m^2 , see Figure A.5.

- L = the length of the measured section along core or line.
- f_i = the interval factor given in Table A3; its ratings have been determined by Palmström (1995) from trial and error of various angles and joint densities.

In practice, each joint is multiplied by the value of (f_i) for the actual angle interval. After some training it should be possible to quickly determine the intervals in Table A3 for the angle δ_i . The intervals chosen

removes the strong influence of the smallest angles, i.e. angles parallel or nearly parallel to the observation plane or bore hole.

angle δ_i	factor f _i	
> 60°	1	
31 - 60°	1.5	
16 - 30°	3.5	
< 16°	6	

Table A3. Selected intervals of the angle (δ_i) and the corresponding factor $(f_i = 1/\sin \delta_i)$.

As the weighted joint density method reduces the inaccuracy caused by the orientation of the observation surface or bore hole, it leads to a better characterization of the rock mass, which in turn may result in a reduced amount of bore holes required in an investigation.

The weighted joint density is approximately equal to the volumetric joint count, i.e. $Jv \approx wJd$

A9. Methods to Find an Equivalent Block Volume Where Joints Do Not Delimit Blocks

According to Section A1, a minimum of three joint sets in different directions are theoretically necessary to delimit blocks in a rock mass. There are, however, cases with irregular jointing where blocks are formed mainly from random joints, and other cases where the blocks are delimited by one or two joint sets and additional random joints. In cases where the jointing is composed of one or two joint sets with no or few random joints, the joints do not define individual blocks. In such cases an *equivalent block volume* is used in the calculations. Such block volume may be found from one of the following methods:

1. Where only *one joint set* occurs, the equivalent block volume may be considered to be similar to the area of the joint plane⁵ multiplied by distance between the two joints: $Vb = L^2 \times S$ Here L is the joint length and S is the spacing between the joints.

(Example: For foliation partings with lengths L = 0.5 m to 2 m and joint spacing S = 0.2 m, the equivalent block volume will vary between $Vb = S \times L^2 = 0.2 \times 0.5^2 = 0.05 \text{ m}^3$ and $Vb = 0.2 \times 2^2 = 0.8 \text{ m}^3$)

- 2. For *two joint sets* the spacing for the two sets (S1 and S2) and the length (L) of the joints can be applied: $Vb = S1 \times S2 \times L$
- 3. For most cases the equivalent block volume can be found from eq. (A-7a): $Vb = \beta \times Jv^{-3}$ which requires input from the block shape factor (β). β can be estimated from eq. (A-9):⁶ $\beta = 20 + 7 \text{ a}/a1$ where a1 and a3 are the shortest and longest dimension of the block.

A method to arrive at a better estimate of β using the length and spacing of the joints, is given in the following:

Eq. (A-9) was developed for three joint sets. Where less than three sets occur, it can be adjusted by a factor n_j , which represents a rating for the actual number of joint sets, to characterize an *equivalent* block shape factor:

$$\beta = 20 + 7 (S_{max}/S_{min})(3/n_j) = 20 + 21(S_{max}/S_{min} \times n_j)$$
 eq.(A-14)

The ratings of n_j are given as:

⁵ Here is assumed that the joint plane is circular, i.e. $A = \pi \cdot L^2/4 \approx L^2$

⁶ As the volumetric joint count can be measured also where joints do not delimit defined blocks, this approach can be applied where few joints sets are found.

3 joint sets + random	$n_{i} = 3.5$
3 joint sets	3
2 joint sets + random joints	2.5
2 joint sets	2
1 joint set + random joints	1.5
1 joint set only	1

4. For small discontinuities (fissures, partings and small joints) for which the lengths can be measured or easily estimated, the length and spacing of the joints correspond to the longest and shortest block dimension, hence the ratio length/spacing = L/S can be applied in eq. (A-14):

eq. (A-15)

 $\beta = 20 + 21 \text{ L} / (\text{S} \times \text{n}_{\text{i}})$

For long joints it is often sufficiently accurate to use a length L = 4 m.

Example: For one joint set $(n_j = 1)$ spaced at S1 = 0.2 m having an average joint length L1 = 2 m, the block shape factor according to eq. (A-15) is $\beta = 20 + 21 \text{ L1/}(S1 \times n_j) = 230$. The volumetric joint count for this set is Jv = 1/S1 = 5. This gives $Vb = \beta \times Jv^{-3} = 1.84 \text{ m}^3$ (For a defined block limited by 3 joints sets crossing at right angles with spacings S1, L1, L1, the volume is $Vb = 0.2 \times 2 \times 2 = 0.8 \text{ m}^3$)

A10. Summary

Measuring block volume instead of the density of joints in the field is often easier and more accurate, especially when the volume can be found from direct observation. Figure A6 shows a summary of the various methods of estimating the block volume described in this Appendix. The methods are based on the main measurements to determine joint density or degree of jointing. The correlations are indicated in Table A4.



Figure A6. The principles in estimating the block volume from various types of joint density measurements.

	TYPE OF OBSERVATION or MEASUREMENT					
	3-D observations	2-D observations	1-D observations			
count	Average joint spacing (S) for each set: $Jv = \Sigma (1/S_i)$	Average joint spacing (S_m) : $Jv = S_m / ka$	Average length of core pieces (fracture intercept, Fi): $Jv = Fi / kl$			
ic joint (v)	or where also random joints (Nr) ^{**)}	Average joint frequency (Na): $Jv = Na \times ka$	Average joint frequency measurement (Nl): $Jv = Nl \times kl$			
olumetr (Jy	occur: $\mathbf{J}\mathbf{v} = \Sigma (1(\mathbf{S}_i) + \mathbf{N}\mathbf{r}/5)$	Weighted 2-D joint density measurement (wJd): Jv = wJd	Weighted 1-D joint density measurement (wJd): Jv = wJd			
The v			Rock quality designation, RQD: Jv = 35 - RQD/ 3.3			
(q/	Average joint spacing (S1, S2, S3) of 3 sets: *) $Vb_0 = S1 \times S2 \times S3$	Weighted joint density measurement (wJd): $Vb_0 = \beta \times wJd^{-3}$	Weighted 1-D joint density measurement (wJd): $Vb_0 = \beta \times wJd^{-3}$			
olume (V	Volumetric joint count (Jv): $Vb_0 = \beta \times Jv^{-3}$	Average joint frequency (Na): Vb $\approx \beta$ (Na \times ka) ³	Average joint frequency (Nl) or spacing (Nl = 1/S) measurement: Vb $\approx \beta$ (Nl × kl) ⁻³			
The block vc			Rock quality designation, RQD: $Vb_0 \approx \beta (35 - RQD/3.3)^{-3}$			
	Direct measurement in situ: Vb = measured volume of the block	Direct measurement in situ: Vb = volume of block estimated from the jointing pattern	Direct measurement in drill cores (of fragments of core diameter or less): Vb = volume of core fragments			
Com	nents:					

Transitions between various types of joint density observations Table A4.

*' Vb_o = block volume for joints or block faces intersecting at right angles. For intersections at other angles the volume can be found from: Vb = Vb_o / sinγ1 × sinγ2 × sinγ3
 (γ1, γ2, and γ3 = angles between the joint sets or between the block faces)
**' Nr = the number of random joints observed within the observation area.
 (A more precise measurement of Nr can be made as is shown for Na in eq. (A-2))

 β = block shape factor; it may be estimated from β = 20 + 7 a3/a1

(a3 and a1 are longest and shortest block dimension respectively)

 $\beta = 27$ to 50 for equidimensional to slightly long or flat blocks, $\beta = 50$ to 150 for most flat or long blocks. Values of 2-D and 1-D correlation coefficients: ka = 1 to 2.5 (average ka = 1.5); kl = 1 to 7 (average kl = 2)